

$$Q: \int \underbrace{y \left(1 + \frac{1}{x}\right) + \cos y}_{M} dx + \int \underbrace{x + \log x - x \sin y}_{N} dy = 0$$

check whether it is exact or not i.e. verify $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$$\left. \begin{aligned} \frac{\partial M}{\partial y} &= \frac{\partial}{\partial y} \left[y \left(1 + \frac{1}{x}\right) + \cos y \right] = 1 + \frac{1}{x} - \sin y \\ \frac{\partial N}{\partial x} &= \frac{\partial}{\partial x} \left[x + \log x - x \sin y \right] = 1 + \frac{1}{x} - \sin y \end{aligned} \right\} \text{exact}$$

$$A = \int_{y=c} M dx = \int y \left(1 + \frac{1}{x}\right) + \cos y dx =$$

$$= y(x + \log x) + x \cos y$$

$$B = \int_{x=c} N dy = xy + y \log x + x \cos y$$

$$\text{Sol: } A + B = K$$

$$xy + y \log x + x \cos y = K$$

Method VI Eqⁿ reducible to exact diff. eqⁿ.

$M dx + N dy = 0$ but $M_y \neq N_x$ so D.E. is not exact

To make such a D.E. exact, sometimes, it is possible to find a multiplier which makes it exact & this multiplier is called Integrating factor (I.F.)

i) If $\frac{M_y - N_x}{N} = f(x)$ "only" then I.F. = $e^{\int f(x) dx}$

ii) If $\frac{N_x - M_y}{M} = f(y)$ "only" then I.F. = $e^{\int f(y) dy}$

• • • — (1)

$$\text{Q: } (2x^2 + y) dx + (x^2 y - x) dy = 0 \quad \text{--- (1)}$$

$$M_y = \frac{\partial M}{\partial y} = 1 \quad N_x = \frac{\partial N}{\partial x} = 2xy - 1$$

$$M_y - N_x = 1 - (2xy - 1) = 2 - 2xy = 2(1 - xy)$$

$$\frac{M_y - N_x}{N} = \frac{2(1 - xy)}{x^2 y - x} = \frac{2(1 - xy)}{x(xy - 1)} = -\frac{2}{x} = \phi(x) \text{ only.}$$

$$\text{I.F.} = e^{\int -\frac{2}{x} dx} = e^{-2 \log x} = e^{\log x^{-2}} = x^{-2} = \frac{1}{x^2}$$

Multiply (1) with I.F. $\frac{1}{x^2}$

$$\underbrace{\left(2 + \frac{y}{x^2}\right)}_M dx + \underbrace{\left(y - \frac{1}{x}\right)}_N dy = 0$$

$$A = \int M dx = \int \left(2 + \frac{y}{x^2}\right) dx = 2x - \frac{y}{x}$$

$$B = \int N dy = \int \left(y - \frac{1}{x}\right) dy = \frac{y^2}{2} - \frac{y}{x}$$

$$\text{Sol: } A + B = K \quad 2x - \frac{y}{x} + \frac{y^2}{2} = K$$

$$\text{Q2) } (2xy^4 e^y + 2xy^3 + y) dx + (x^2 y^4 e^y - x^2 y^2 - 3x) dy = 0$$

$$\frac{\partial M}{\partial y} = 2x [y^4 e^y + e^y \cdot 4y^3] + 2x \cdot 3y^2 + 1$$

$$= 2xy^4 e^y + 8xy^3 e^y + 6xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 2xy^4 e^y - 2xy^2 - 3$$

$$M_y - N_x = 8xy^3 e^y + 8xy^2 + 4 = 4(2xy^3 e^y + 2xy^2 + 1)$$

$$M_y - N_x =$$

$$M_y - M_x = 0xy^3e^y + 0xy^4 + 4 = 4(2xy^3e^y + 2xy^2 + 1)$$

$$\frac{M_y - M_x}{M} = \frac{4(2xy^3e^y + 2xy^2 + 1)}{y(2xy^3e^y + 2xy^2 + 1)} = \frac{4}{y} = \phi(y)$$

$$\frac{M_x - M_y}{M} = -\frac{4}{y} = \phi(y)$$

$$-\int \phi(y) dy = -\int \frac{4}{y} dy = -4 \log y = \log y^{-4}$$

$$\text{I.F.} = e^{-4 \log y} = \frac{1}{y^4}$$

multiplying (2) with I.F.

$$(2xe^y + \frac{2x}{y} + \frac{4}{y^3}) dx + (x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}) dy = 0$$

$$A = \int M dx = \int (2xe^y + \frac{2x}{y} + \frac{4}{y^3}) dx$$

$$= x^2e^y + \frac{x^2}{y} + \frac{4x}{y^3}$$

$$B = \int N dy = \int (x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}) dy$$

$$= x^2e^y + \frac{x^2}{y} + \frac{x}{y^3}$$

Sol. $A + B = C$ $x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = C$ Ans.